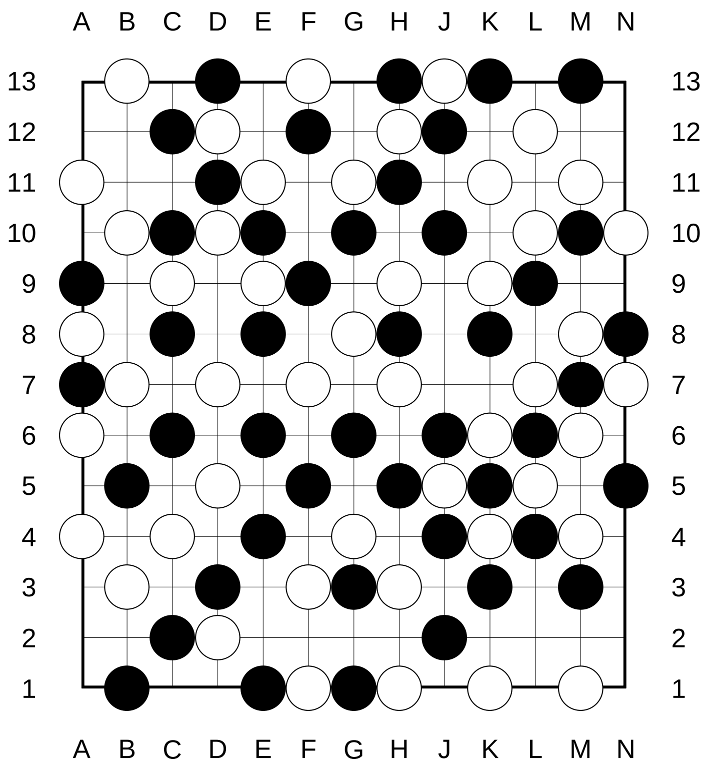
Antwerp sprouts

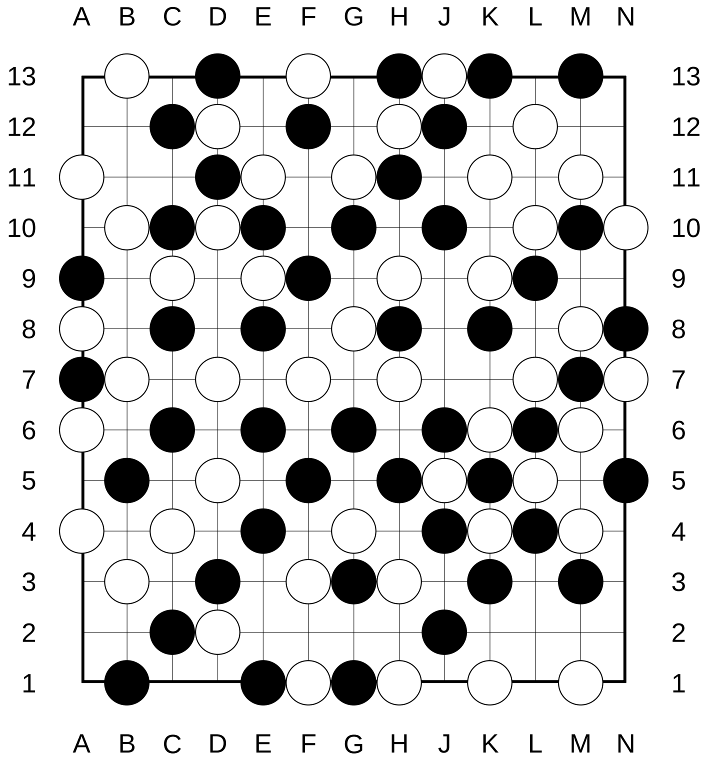
lieven le bruyn

2004

university of antwerp



Een COLgo positie.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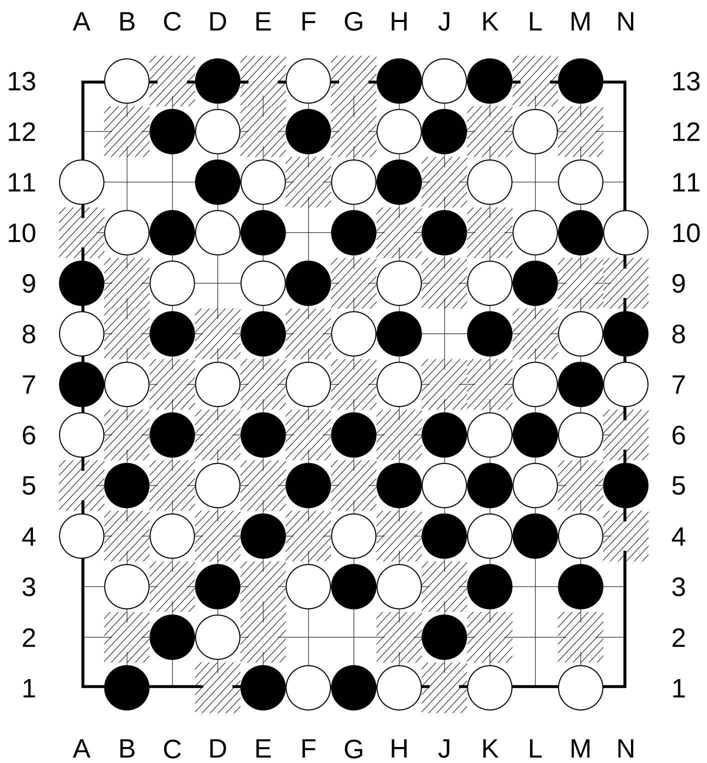
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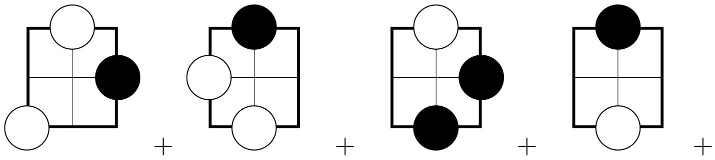
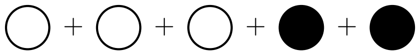
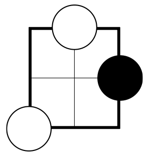
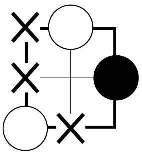
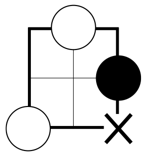


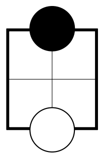




$\{$  $|$  $\} = 1 + 1 - 1 = 1$

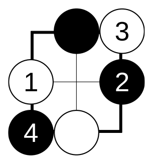
$$G = \{ \, G^L \mid G^R \, \}$$

$$\begin{array}{ll} G > 0 & \text{asa Links heeft winnende strategie} \\ G < 0 & \text{asa Rechts heeft winnende strategie} \\ G = 0 & \text{asa 2de speler heeft winnende strategie} \\ G || 0 & \text{asa 1ste speler heeft winnende strategie} \end{array}$$



$= 0$

want 2de speler heeft winnende 'spiegel-strategie', bvb.

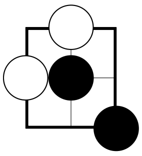
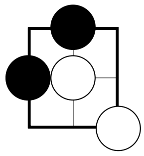


Tweedledee-Tweedledum strategie

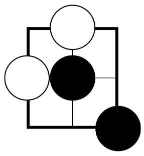
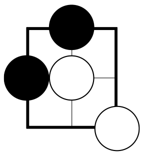
Ieder spel G heeft een 'negatief' $-G$ zodat

$$G + (-G) = 0$$

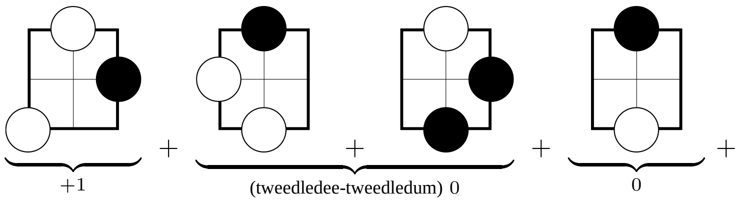
Verwissel alle kleuren in $-G$. Bvb.

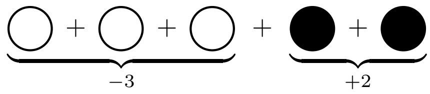
$G =$  dan is $-G =$ 

want

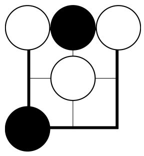
 $+$  $= 0$

(kopie-strategie).


+1
(tweedledee-tweedledum) 0
0

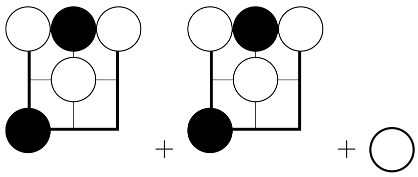

−3
+2

Niet-gehele voordelen bestaan

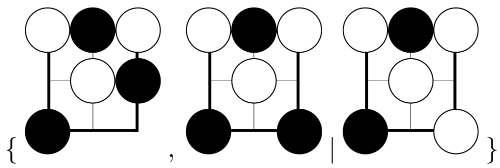


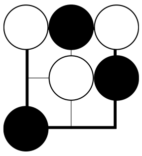
$= \frac{1}{2}$

bewijs 1 (strategie)

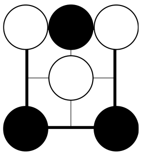


bewijs 2 (recursie)

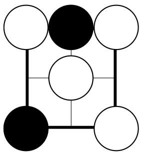




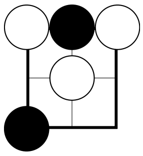
$= \{\emptyset|0\} = -1$



$= 0$



$= \{0|\emptyset\} = 1$



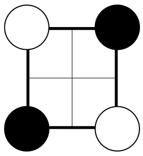
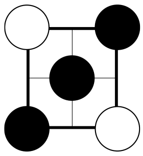
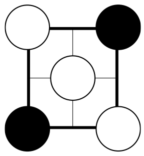
$= \{\ -1, 0 \mid 1\ \} = \frac{1}{2}$

Berlekamp's simplicity rule

$G = \{G^L \mid G^R\ \}$ is *getal* asa alle opties in G^L en in G^R getallen zijn en voor alle opties geldt $G^L < G^R$.

Dan is de waarde van G het eenvoudigste dyadische getal $> G^L$ en $< G^R$.

Niet-getal posities bestaan.

 $= \{$  $|$  $\} = \{0|0\} = *$

$$*||0$$

Hoofdstelling voor COLgo posities : De waarde van iedere COLgo positie is van de vorm

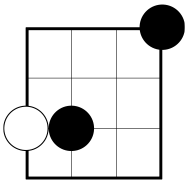
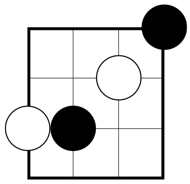
$$g \qquad \text{of} \qquad g + *$$

met g een (dyadisch) getal is. Links (bLack) wint als $g > 0$, Rechts (white) als $g < 0$. Als $g = 0$ dan wint de tweede speler tenzij $g + *$ dan wint de eerste speler.

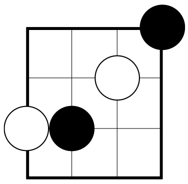
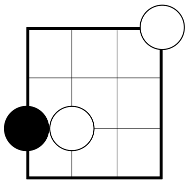
bewijs : Neem een COLgo positie $G = \{G^L \mid G^R\}$. We beweren :

$$G^L + * \leq G \leq G^R + *$$

Bvb.

$G =$  en $G^R =$ 

Dan wint Links als tweede speler $G^R - G + *$,d.i. $G^R + * - G \geq 0$

 $+$  $+ \{$  $\}$

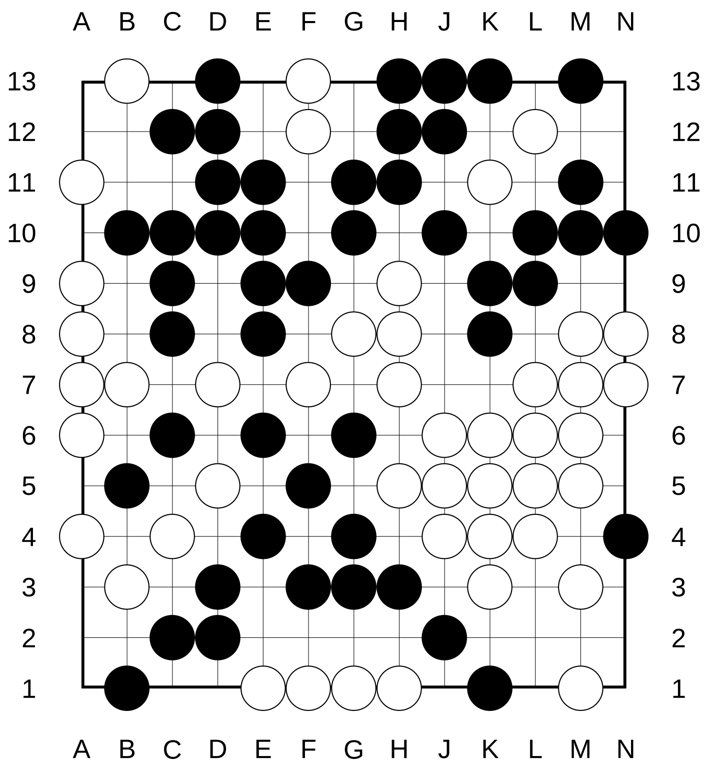
Analoog bewijs je $G^L + * - G \leq 0$ (Rechts wint als tweede speler).

Recursief is iedere $G^L = g_L$ of $g_L + *$ (analoog voor G^R). Neem :

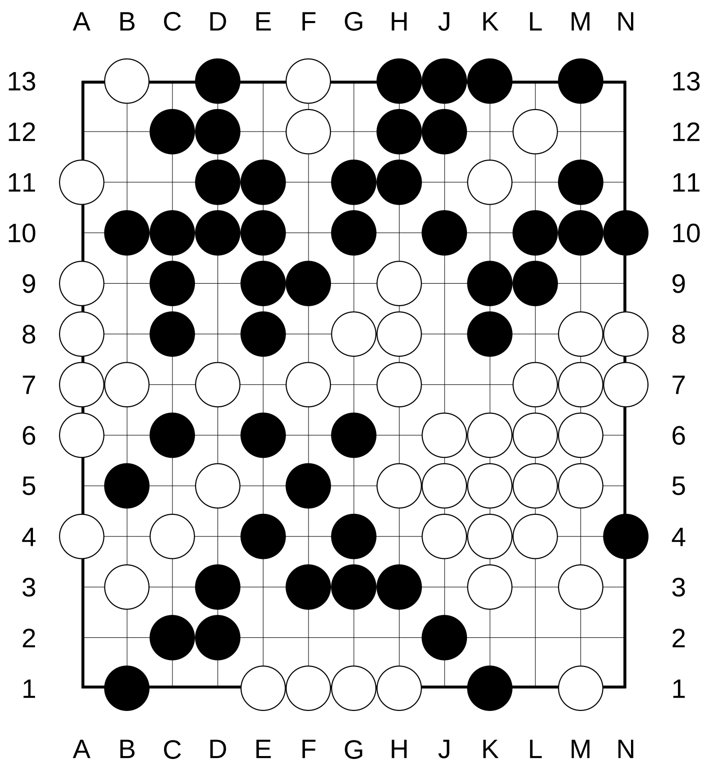
$$g' = max(g_L) \qquad \text{en} \qquad g" = min(g_R)$$

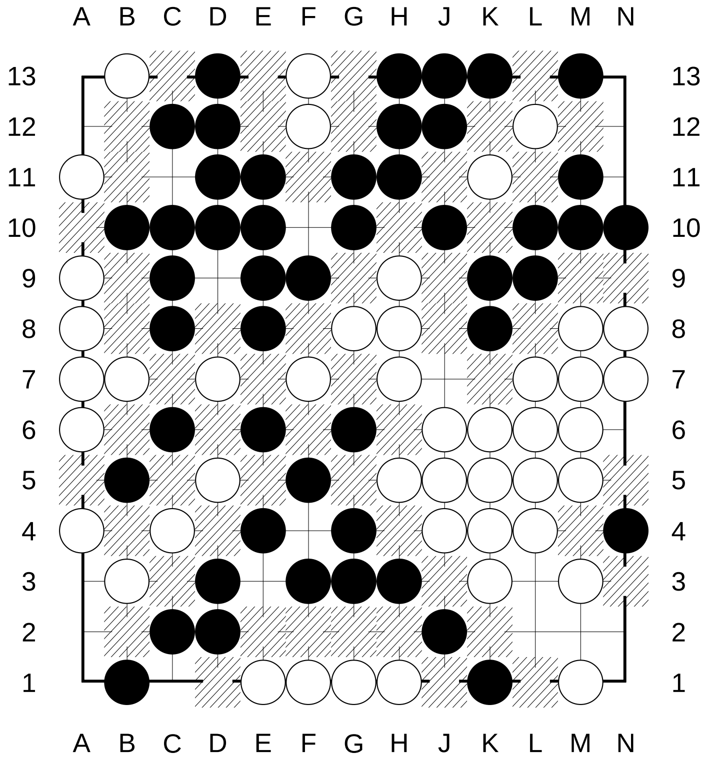
dan hebben we volgende mogelijkheden :

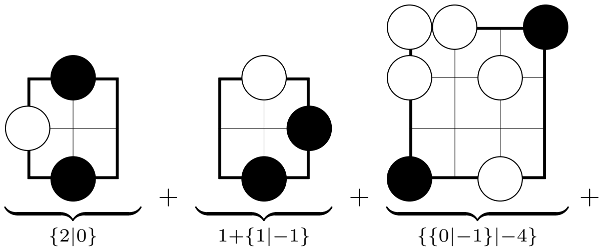
$$\begin{cases} g' < g" \Rightarrow G = g \text{ of} g + * \\ g = g' = g" \Rightarrow G = \{g \mid g\} = g + * \text{ of } \{g + * | g + *\} = g \end{cases}$$

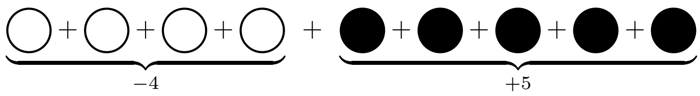


Een SNORTgo positie.




A B C D E F G H J K L M N
13 12 11 10 9 8 7 6 5 4 3 2 1



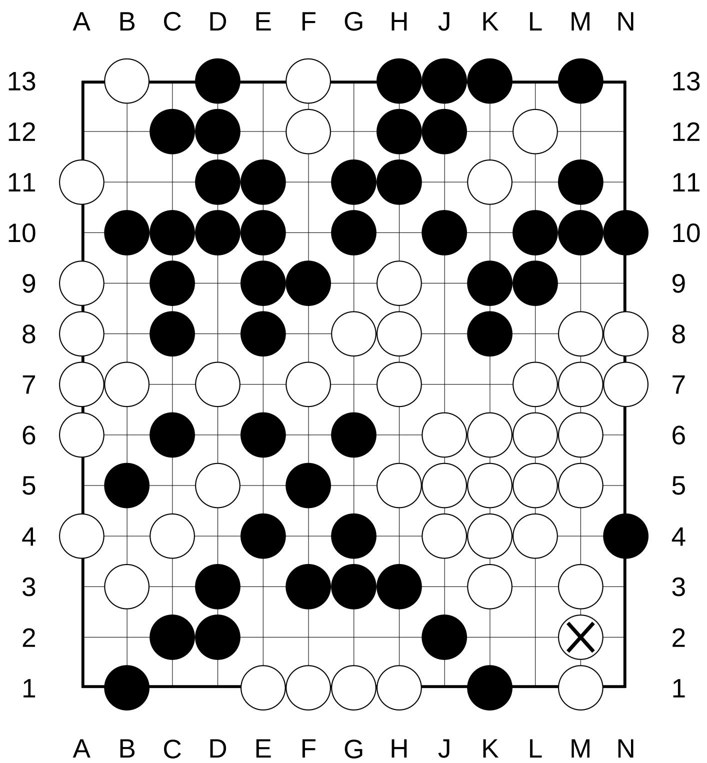


Combinatorial Game Suite : Java Applet

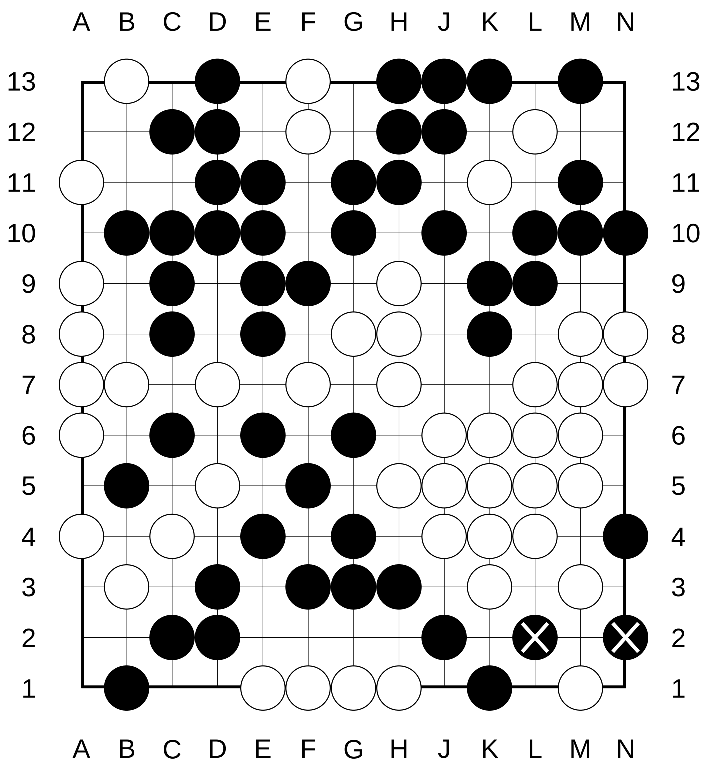
http://cgsuite.sourceforge.net/download.php

$$G = \{\{3|2\}|-1\}||0$$

dus eerste speler heeft een winnende strategie!



De unieke! winnende zet voor Rechts (white).



De winnende opties voor Links (bLack).